Comparative Study of Optimization Methods for Dispatching Unit Generators in a Power Plant

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***Abstract*— Optimization in dispatch plan of a power plant is an important thing because it is closely related to power generation cost. Generating electricity with the least cost is always preferred. In this study an example of determining optimum dispatch plan by comparing three methods by using a set of generation and load sample data is given. The methods used in this study are Priority List, Dynamic Programming, and Lagrange Relaxation. Based on the simulation's result, Both Dynamic Programming and Lagrange Relaxation tend to produces the most efficient generation cost. All simulation and calculation conducted using basic excel and MATLAB software. The study and analysis can be implemented to a real case, so that real efficiency can be expected.**

Keywords—optimization, dispatch, generator, generation, cost, Lagrange, Priority List, efficiency

I. PREFACE

Power plant is a key element in power system because it has main function to generate power to be supplied across the system. Power generated by power plant in a system must to be balance with existing demand level. If it is not fulfilled there will be disturbance or fault at the system mainly related to voltage and frequency.

A power plan may consist of a set of generators operated in parallel. And also a generator may have different properties and specification, especially in fuel consumption characteristic compared to other generators. This characteristic is directly related to required production cost by the generator to provide power to the connected system. Because it is generally expected more efficient power plant, and this efficiency closely related to generation cost, it is expected to operate unit generator in a proper manner and plan.

If a power plan consist of a set of generator, to achieve more efficient way and less cost to generate electricity, it should have a proper plan which generator to operate in a power level at a specific time. This is sometimes called generator dispatch plan. Generally, a generator with the least specific cost to operate has to be prioritized to operate than other. But in real case, there are other aspect that should be considered, such as power generation limit, start-up cost, start-up time, required spinning reserve, and others.

This study is conducted to analyze and to implement some existing methods of dispatch plan optimization. Those methods are: Priority List, Dynamic Programming, and Lagrange Relaxation. Objective of this study is to give example about how to make a dispatch plan that cost optimization-oriented. Also it is expected that general concept and comparison between those methods can be understood.

II. PREVIOUS RESEARCHES

In previous researches and studies, reference [1] mentioned that the unit commitment schedule produced by Lagrange Relaxation technique more cost efficient than Priority List Method. For iteration in the Lagrange Relaxation method, there are no guarantee the duality gap of this method will decrease in next iteration, because the state of unit can change into ON or OFF, so the duality gap is fluctuated. But the purpose of this method is get the smallest difference of primal value and dual value and the convergence can be achieved. In the research reference, the down time constraints, minimum up time, and start-up cost are calculated. But the value of reserve capacity and ramp rate are eliminated and the shut-down cost assumed as zero. Reference [2] mentioned Lagrange Relaxation method gives the solution of unit commitment more efficient. The other benefit of Lagrange Relaxation is the algorithm can get the solution as fast as possible of large power system with the right selection of Lagrange Multiplier.

Reference [3] mentioned there are three different approaches of dynamic programming. For small size power system, conventional dynamic programming is more efficient than other method. But, in the large power system Truncation DP is more efficient than Conventional and sequential DP. Reference [4] mentioned Methodological Priority List is innovation from Priority List Method. In the method, some constraints such minimum time up, minimum time down, start-up cost, spinning reserves are defined as the minimization of total objective function. MPL combined the new algorithm with the result of conventional priority list. So the calculation is faster than conventional priority list.

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Reference [5] mentioned there is a new approaches of priority list that is combined with genetic algorithm. Based on its study mentioned when using hybrid method between genetic algorithm and priority list method in the 26 units system produce a smaller minimum value compared to when just using genetic algorithm or priority list.

Reference [6] mentioned there is a method that combine Lagrange Relaxation and Invasive Weed Optimization. The Lagrange relaxation used to reserve, forecast power demand, and electricity price. Then Invasive Weed Optimization used to calculate the new Lagrange Multiplier based on the duality gap difference of primal value and dual value. After testing in 3 units system and 10 units system, the result showed the method that combine Lagrange Relaxation and Invasive Weed Optimization provide better solution or result than other conventional method or existing method. Reference [7] mentioned combination Lagrange Relaxation and Particle Swarm Optimization. The purpose of the combination is to update the Lagrange Multiplier and improve the performance of Lagrange Relaxation. Based on testing in the 4 units system and 10 units system, the hybrid method is cheaper cost than other method such Lagrange Relaxation, Genetic Algorithm, and Hybrid Particle Swarm Optimization.

III. Basic Theories

There are 3 optimization methods that going to be compared in this study, those are: Priority List (PL), Dynamic Programming (DP), and Lagrange Relaxation (LR). Generation data that will be used are total generator unit in a power plant, maximum and minimum power limit that can be delivered by each generator, and cost function of each generator based on delivered power, or F(P). The cost functions are approached by quadratic equation as in (1).

$$F(P) = a.P^2 + b.P + c \tag{1}$$

As for load data is assumed in step by 1 hour per step in a period of time. Normally the period will be one day or 24 hours, but in this study of comparison the period is set in 8 hours (h). Also because the power plant type is gas and diesel, assumed there is no required dedicated starting time for each generator.

Foundation of economic dispatch optimization is a Lagrange Equation which have to be solved in a certain boundaries. In general, if there are n number of generator which are operated to supply power to a load, or P_{load}, and each generator have operation limit $P_{i,min}$ and $P_{i,max}$, a set of function that have to be solved is show in (2) and (3).

$$\mathcal{L}(P_1, \dots, P_n, \lambda) = \sum F_i(P_i) - \lambda\left(\sum P_i\right) \tag{2}$$

$$\frac{\partial F_i(P_i)}{\partial P_i} = \lambda \,;\, P_{i,min} \le P_i \le P_{i,max} \tag{3}$$

for each generator i, and (4)

$$\sum P_i = P_{load} \tag{4}$$

In the Lagrange Equation and the first derivative assumed that each generator is committed and operated, or turned ON and there is any generation limit exceeded to meet each level of demand.

In this study a combined diesel-gas power plant that operated to fulfill the load consist of four generators with each limit, cost function, and start-up cost (SUC) data available in Table I below. Furthermore, relation between generated power, production cost, and operational limit for each generated showed in Figure 1.

TABLE I. A Set Of Generator Data In A Diesel-Gas Power Plant

Unit No	P min (MW)	P max (MW)	a ($/MW²h)	b ($/MWh)	c ($/h)	SUC ($)
1	8	32	0.515	10.86	500	60
2	17	65	0.227	8.341	300	240
3	35	150	0.082	9.9441	100	550
4	30	150	0.074	12.44	388.9	550

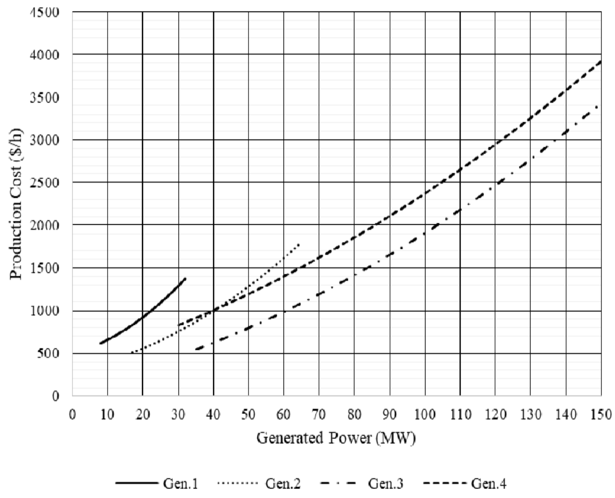

Fig. 1. Relation Between Generator Power, Production Cost, And Operation Limit

A set of demand sample consist for a set of one hour with total period 8 hours as shown in Table II. Each demand level is unique, and the demand pattern is fluctuating. Each demand level also still within range of total generation limit, whether minimum or maximum. Assumed that all demand is met by total generation of the power plant.

TABLE II. Load Data Of 8-Hour Period

Time (h)	Load Demand (MW)
1	168
2	150
3	260
4	275
5	313
6	347
7	308
8	231

A. *Priority List Method*

Priority List (PL) Method one of the simplest and easiest procedure to determine dispatch pattern optimization of a set of generator. In this method, generation dispatch sequence determined by each average generation cost when the generator operated in maximum limit in a period or called Full Load Cost (FLC), here in \$/MWh.

My PL dispatch pattern determined by using generator with cheapest FLC at first and prioritize it to operate. When the load level exceed the generator maximum limit, the next generator with second cheapest FLC turned on and operated. The procedure goes on until all generators turned on if required.

Furthermore, in PL optimization algorithm there are some parameter must be considered, those are: spinning reserve required to meet demand forecast, minimum shut-down time required, and necessity of start-up time.

B. Dynamic Programming Method

Dynamic Programming (DP) Method uses recursive algorithm to calculate and determine minimum cost of generation to meet a set of demand in a period. The cost consist of production cost and transition cost, and the most efficient determined by choosing of generation combination path or way. For example, a generation cost in hour K by combination I calculated by (5).

$$F_{cost}(K,I) = \min[P_{cost}(K,I) + S_{cost}(K-1,L;K,I) \\ + F_{cost}(K-1,L)] \quad (5)$$

Where $F_{cost}(K,I)$ is minimum cost to 'arrive' at state (K,I); P_{cost} (K,I) is production cost at state (K,I), and S_{cost} (K-1,L:K,I) = transition cost from state (K-1,L) to arrive at state (K,I).

Overall DP recursive algorithm used to determine generation dispatch optimization shown in Figure 2.

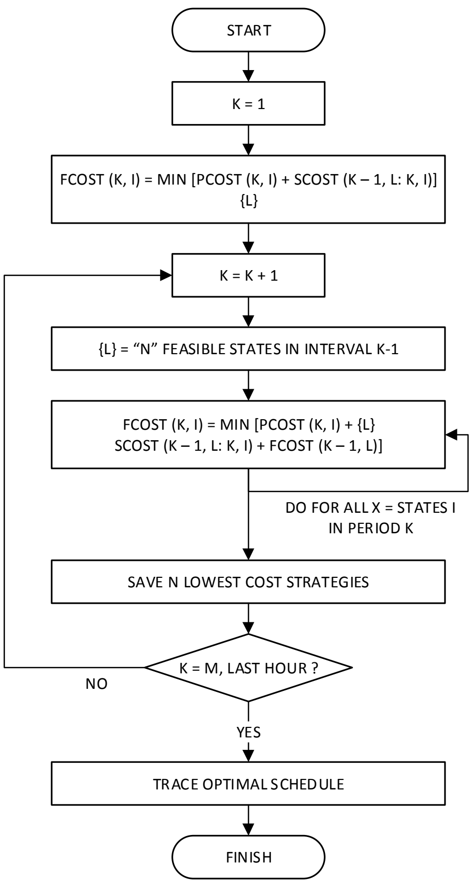

Fig. 2. Dynamic Programming Algorithm For Dispatch Optimization [8]

State (K,I) is the I-th combination at hour K. In DP, one or more strategies or paths selected from a state to the next state, until all period is covered. The last hour is reached, those paths is re-evaluated by considering required transition cost, and the total generation cost of all selected ways is compared to determine the cheapest one. As of X is number of state that available in a period, and N is number of strategies or paths that can be saved for each step.

C. Lagrange Relaxation Method

Lagrange Relaxation (LR) Method is an optimization method based of Lagrange Function showed in (6).

$$\mathcal{L}(x,y,\lambda) = f(x,y) - \lambda \cdot g(x,y) \quad (6)$$

In LR defined variable U_i^t which equal to zero (0) if generator i is turned off at time t, and equal to one (1) if the generator is turned on (operated) at that time. The load which have to be met is considered as a constraint, beside each generation operating limit. In general, the complete equation of LR is showed in (7).

$$\mathcal{L}(P,U,\lambda) = F(P_i^t, U_i^t) + \sum_{t=1}^{T} \lambda^t \left(P_{load}^t - \sum_{i=1}^{N} P_i^t U_i^t\right) \quad (7)$$

LR is an iterative method which separate the coupling between objective cost function and load constrain. In each iteration step, specific production cost or lambda (λ) for each time step is determined and each generator is evaluated to it by second derivative basic Lagrange Function in (8).

$$\frac{\partial F_i(P_i)}{\partial P_i} = \lambda \quad (8)$$

It the P_i is exceed generator minimum value, Uit is set to zero (OFF) otherwise it set to one (ON). Then the total lambda based generation is calculated. Also dual value or q* (λ) is determined as in (9).

$$q^*(\lambda) = \sum_{i=1}^{N} \min \sum_{t=1}^{T} \left\{ \begin{matrix} [F_i(P_i^t) + Startup\ Cost_i^t] U_i^t \\ -\lambda^t P_i^t U_i^t \end{matrix} \right\} \quad (9)$$

Based on previously determined of each unit commitment U_i^t, the primal value J* is calculated in (10). Here in each time step, the generator included in the calculation only if U_i^t is equal to 1.

$$J^* = \mathcal{L}(P,\lambda) = F(P_i^t) + \sum_{t=1}^{T} \lambda^t \left(P_{load}^t - \sum_{i=1}^{N} P_i^t\right) \quad (10)$$

Then the value of lambda at each time step is adjusted by (11) to (13).

$$\lambda^t = \lambda^t + \left[\frac{d}{d\lambda} q(\lambda)\right] \alpha \quad (11)$$

$$\alpha = 0.01\ when \frac{d}{d\lambda} q(\lambda)\ is\ positive \quad (12)$$

$$\alpha = 0.002\ when \frac{d}{d\lambda} q(\lambda)\ is\ negative \quad (13)$$

Because LR is an iterative method, there will be no exact solution. Here comparison of q* and J* or called "duality gap" is used whether the result is already acceptable or not, as in (14).

$$duality\ gap = \frac{J^* - q^*}{q^*} \quad (14)$$

Here is complete algorithm of LR in generator dispatch optimization shown in Figure 3.

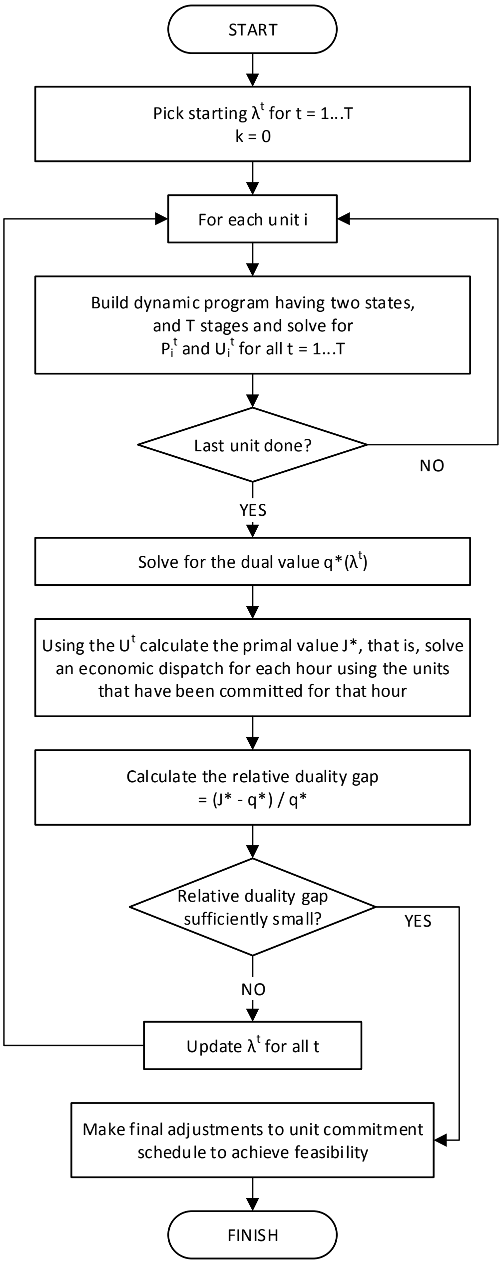

Fig. 3. Lagrange Relaxation Algorithm For Dispatch Optimization [8]

IV. Simulation Result

In the following sections the implementation results of each dispatch optimization method are shown.

A. *Priority List Result*

Here is the generators sorted by FLC properties from the cheapest to the most expensive as shown in Table III.

TABLE III. GENERATOR UNIT SORTED BASED ON FLC

Unit No.	3	4	2	1
FLC ($/MWh)	22.91077	26.13267	27.71138	42.965

Then a list of combinations based on FLC order is made, from only one generator operated which have the cheapest FLC to a combination that consist of all generators operate. The minimum and maximum production limits from each combination are also showed as in Table IV.

TABLE IV. COMBINATION OF GENERATOR OPERATION BASED IN PRIORITY LIST FLC

Code	**Combination/ generator which operate**	**Min MW**	**Max MW**
A	unit 3	35	150
B	unit 3+4	65	300
C	unit 3+4+2	82	365
D	All generators operate	90	397

After the priority list is made then load fulfillment is simulated. A load fulfillment with minimum generator operating is prioritized. When two or more generator have to operate, each power dispatch levels are calculated by standard Lagrange optimization for generation as in (15). If a generator is initially on then switched on, then Start-Up Cost is included in total cost calculation. Finally, calculation result of generator power dispatches is shown in Table V.

$$\frac{\partial F_i(P_i)}{\partial P_i} = \lambda\, ; P_{i,min} \leq P_i \leq P_{i,max} \quad (15)$$

TABLE V. CONTRIBUTION FROM EACH GENERATOR FOR HOURLY LOAD BASED ON PL OPTIMIZATION

hour	**Load (MW)**	**Combination**	**P1 (MW)**	**P2 (MW)**	**P3 (MW)**	**P4 (MW)**
1	168	unit 3+4	0	0	88	80
2	150	unit 3+4	0	0	79	71
3	260	unit 3+4	0	0	131	129
4	275	unit 3+4	0	0	138	137
5	313	unit 3+4+2	0	51	132	130
6	347	unit 3+4+2	0	56	146	145
7	308	unit 3+4+2	0	51	130	127
8	231	unit 3+4+2	0	39	99	93

From the dispatching plan in Table V, total generation cost in a period can be calculated. By assuming no cost is needed for a generator when it in "OFF" or zero state and if all start-up costs are neglected, then total generation cost for 8 hours **$47,303.48**. If the start-up costs are taken into account, then total generation cost in the whole period is **$47,543.48**.

B. *Dynamic Programming Result*

Dynamic Programming (DP) method is implemented by listing every possible combination of generator commitment (1 or 0) and sorting it based on maximum power that can be delivered (P_{max}) by each combination. Here if there are n number of generators, then there are 2^n-1 possible combination that could be made, by excluding if all generators are turned off. From existing generator data, result of sorting process is shown in Table VI.

Combination	Production Cost per hour ($/h)							
15	3971.08	3597.55	6195.94	6608.75	7717.39	8785.69	7566.37	5437.63
7 *Path B*	3527.54	3142.53	5831.44	6260.37	7413.88	8527.25	7256.62	5044.54
11	3885.94	3476.81	6343.00	6801.65	8036.42		7867.98	5502.42
3 *Path A*	3467.77	3042.52	6034.96	6515.95				5154.67
14	4042.03	3559.46						6006.23
13	4507.13	4014.15						6489.52
6	3697.26	3181.11						
5	4170.67	3645.45						
10	4287.874	3701.717						
9	4770.59	4179.88						
2		3436.62						
1		3919.90						
hour	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**
Load (MW)	**168**	**150**	**260**	**275**	**313**	**347**	**308**	**231**

Fig. 4. Optimization And Calculation Result Using Dynamic Programming Method

TABLE VI. EVERY POSSIBLE COMBINATION OF GENERATOR COMMITMENT

Code	P1	P2	P3	P4	Pmax (MW)
15	1	1	1	1	397
7	0	1	1	1	365
11	1	0	1	1	332
3	0	0	1	1	300
14	1	1	1	0	247
13	1	1	0	1	247
6	0	1	1	0	215
5	0	1	0	1	215
10	1	0	1	0	182
9	1	0	0	1	182
2	0	0	1	0	150
1	0	0	0	1	150

The load pattern varies between 150 MW to 347 MW, so every combination are possible to be used. To choose which combination of generator commitment is used in a load level, the load and cost data of every possible combination are shown as in Figure 4.

Production cost is of a combination in an hour calculated by standard Lagrange optimization for >1 generator. Note that the gray areas with no value mean the combination cannot fulfill the load demand at the period.

Then for each hour the cheapest production is selected, here by light gray highlight so it form a 'path'. From hour 1 to hour 8, the path passes combination 3-3-7-7-7-7-7-7.

Total production cost of path A, is **$46,844.38**, neglecting start-up cost. If the start-up cost is included, then total generation cost is $47,084.38. If the available pathways in Figure 4 are analyzed further, all-7 path, or path B, give cheaper total generation cost which is **$47,004.17**. So if start-up cost is included in cost calculation, dispatching the generator set following path B is more efficient than as in path A.

C. Lagrange Relaxation Result

The result of 25th optimization iteration using Lagrange Relaxation (LR) method is shown in Table VII.

TABLE VII. RESULT OF 25TH ITERATION USING LAGRANGE RELAXATION OPTIMIZATION

Hour (h)	Load (MW)	P1 (MW)	P2 (MW)	P3 (MW)	P4 (MW)	Cost ($/h)
1	168	0	0	88	80	3467.77
2	150	0	0	79	71	3042.52
3	260	0	44	111	106	5831.44
4	275	0	46	117	113	6260.37
5	313	0	51	132	130	7413.88
6	347	0	56	146	145	8527.25
7	308	0	51	130	127	7256.62
8	231	0	39	99	93	5044.54
Total cost ($)						**46844.38**

Criteria of iteration convergence is determined based on deviation between primal value (J*) and dual value (q*), which called dual relativity gap. The whole iteration process is convergent as shown in Figure 5. In the 25th iteration, the gap already can be accepted (<0.03%), so the value can be taken as a result. Here J* **is $46,844.38** and q* is **$46,831.05**.

Based on the iteration result, total generation is equal to J* or **$46,844.38**. Note that in this Lagrange Relaxation iteration, the start-up cost is neglected. If start-up cost is included, a simple way is by directly adding it to previous result. Here the needed start-up cost is only for unit generator 2 as in transition from hour 2 to hour 3. So the total generation cost becomes **$47,084.38**.

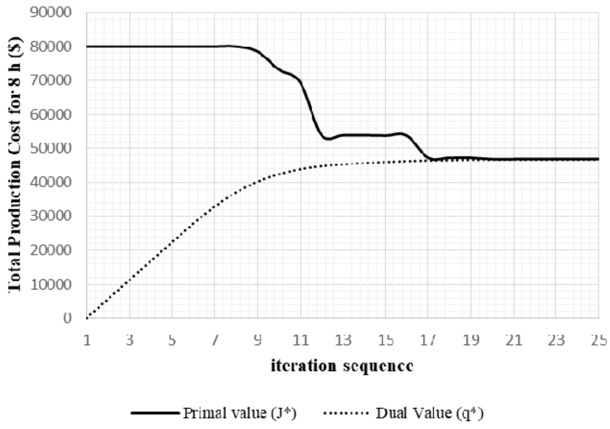

Fig. 5. Iteration Convergence Of Primal And Dual Value

V. CONCLUSION AND FURTHER STUDY

Calculation results of total generation cost for 8 hours from all three methods are recapitulated as in Table VIII.

TABLE VIII. RECAPITULATION OF DISPATCH OPTIMIZATION RESULT

Method	Total Generation Cost ($) for 8-h period	
	Neglecting Start-up cost	including Start-up cost
PL	47303.48	47543.48
DP	46844.38	47004.17
LR	46844.38	47084.38

Based on Table VIII it is known that DP and LR methods both give same result of cheapest generation cost for an 8 hours period which are $46,844.38, if start-up cost is neglected. If the cost is considered, then the DP method give a better result which is $47,004.17 compared to LR's $47,084.38. Also know that for both cases, PL method always gives the most expensive generation cost, or the furthest from the most optimum result.

Based on the result of the study here can be concluded that DP and LR methods can calculate more efficient generation cost compared than PL. It can be profitable if management of power plant reconsider and reanalyzed it is generators dispatch plan using those two methods, particularly if the existing plan is already calculated using PL method.

For further study, there are some aspect that could be improved, including:

- The generator cost function and load pattern may be based on existing or real data.
- Steps of load level can be made less than per hour, so the result can be more precise.
- Reconsidering start-up and minimum off period for each generator, especially if modelling and optimizing dispatch plan for heat based power plant which has more significant time to start and be shut down than diesel or gas engine power plant.
- In Lagrange Relaxation, transition cost of an OFF-then-turned ON generator may have to be included in each iteration process rather than at the very end. By this way the method may give a better result of optimization compared to other methods.

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